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ANALYSIS OF CONSTRAINED-LAYER DAMPING OF FLEXURAL AND EXTENSIONAL WAVES IN INFINITE, FLUID-LOADED PLATES

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ABSTRACT

This study contains a mathematical analysis of constrained-layer damping (CLD) in plates of infinite extent, with an emphasis on the physical understanding of some special features, including fluid loading. Previous work is expanded to cover extensional waves. Some essential aspects of fluid loading may be understood by applying thin-plate theory. Therefore thin-plate theory of extensional waves in a fluid-loaded plate was developed as a counterpart to that for flexural waves. The description and examples of CLD follow three models: the first is an extension of Kerwin's 1959 model, the second is a hybrid model for which the base plate is treated by exact elasticity theory, and finally a fully exact model for all three layers. Examples and comparisons are given.

NOMENCLATURE

c_p	*extensional wave speed* for thin base plate
c_d	dilatational wave speed in base plate material
c_3	extensional wave speed for thin constraining layer
c_o	wavespeed in medium
d_i	$= h_i/2$; h_i-layer thickness
E_i	Young's modulus for three layers
E_i^e	extensional modulus for three layers; $E^e = E/(1-\nu^2)$
F	fluid-loading parameter
i	index of layers: 1-base plate, 2-elastomer, 3-constraining layer
k	wavenumber in plate, $k = k' - i\alpha$
k_o	wavenumber in medium
q^s	$= \sigma_+ + \sigma_-$
w	displacement perpendicular to plate
α	attenuation coefficient
ν_i	Poisson's ratio for three layers
ρ_i	density of three layers

G_2 complex shear modulus of elastomer, $G_2 = G_2'(1+i\beta)$

g complex shear parameter, $=G_2/(k'^2 E_3^e h_2 h_3)$, $g=g'(1+i\beta)$

r =3/4 for flexural, =1/2 for extensional waves

ρ_o density of medium

$\sigma_{+,-}$ stresses at opposite sides of plate

τ $= [(k/k_o)^2 -1]^{1/2}$

ω_c characteristic frequency for extensional waves in fluid-loaded plate

Ω dimensionless frequency, $\Omega = \omega/\omega_c$

INTRODUCTION

The principle of constrained-layer damping of acoustic waves consists of attaching a thin elastomer layer with high viscoelastic loss plus a stiff covering layer to a bar, plate, or structure. This stiff layer forces the elastomer into shear, with concomitant large loss, as compared with purely extensional loss in the elastomer without cover layer. Although the loss tangent of the shear modulus G is almost the same as the loss tangent in the Young's modulus E, the energy loss in the constrained layer is of the order $1/kh_2$ of that in an unconstrained layer.

This physical explanation of the effect is represented in a classical paper by E.M. Kerwin [1]. His model gives good results within the given restrictions: flexural waves at low frequency in a main plate with thin additional layers. Extensional waves are not considered, and fluid loading is not readily introduced.

In order to retain the advantage of Kerwin's model of providing an explicit algebraic expression for the attenuation coefficient in terms of geometric and elastic parameters, the model was extended and compared with more exact formulations, a "hybrid" model and an exact model [2,3].

In this study the analysis is extended to cover extensional waves. Since flexural waves were discussed in previous publications [2,3], only some of their features will be mentioned here. For most combinations of plate and medium, the flexural wave speed in the plate, which increases from zero at zero frequency, reaches a value equal to the speed in the medium, at a certain frequency. Thus one distinguishes radiating and non-radiating ("subsonic") waves in the plate. This phenomenon does not occur for extensional waves in most cases.

HIERARCHY OF MODELS

In the case of flexural waves in a fluid-loaded plate, physical insight was obtained by using thin-plate theory. A parallel thin-plate theory for extensional waves in a fluid-loaded plate is presented. The formalism for extensional waves in terms of the hybrid model and fully exact model is the same as for flexural waves.

The extended Kerwin model includes extensional waves. It is represented by the following expression,

$$\alpha/k' = (r\beta)\,\frac{E_3^e\, h_3}{E_1^e\, h_1}\;\frac{g'[1 - (c/c_3)^2]^2}{[1 - (c/c_3)^2 + g']^2 + \beta^2 g'^2} \tag{1}$$

The following outline lists the three models for constrained layer damping used in the examples, with their characteristic properties.

1. Extended Kerwin model
   a. Flexural or extensional waves
   b. Inertia of constraining layer
   c. Complex shear parameter
   d. Wave speed from thick-plate theory

2. Hybrid model
   a. Exact elasticity theory for base plate
   b. Other two layers as in Kerwin
   c. Flexural and extensional waves
   d. With or without fluid loading

3. Exact model
   a. Exact theory for all layers
   b. Flexural and extensional waves
   c. With or without fluid loading

In all the following examples of the analysis the base plate and additional layers have physical and geometric parameters as listed in table I. The (complex) shear modulus of the elastomer is assumed independent of frequency, in order to emphasize the mechanical aspects of the technique without viscoelastic effects.

THIN-PLATE THEORY FOR EXTENSIONAL WAVES IN FLUID-LOADED PLATE

Thin plate theory for flexural waves in a fluid-loaded plate may be found in Ref.[4]. It appeared difficult to derive an analogous expression for extensional waves directly. Therefore the problem was approached as follows.

One starts from the structural equations for waves in plates obtained from exact elasticity theory by integrating and averaging along the direction perpendicular to the plate. From this, one may derive a thick-plate theory for extensional waves [5]. Fluid loading is represented by the sum of the stresses at both sides of the plate, given as $q^s = \sigma_+ + \sigma_-$. Thin-plate theory follows by dropping terms containing the factor (kd). (In this section all quantities refer to the base plate.) The result is equivalent to adding a fluid-loading term F to the familiar equation for extensional waves in a plate,

$$[E/(1-\nu^2)]\ \partial^2 w/\partial x^2 + F = \rho\ \partial^2 w/\partial t^2, \tag{2}$$

where F is given by

$$F = [1 - (c/c_d)^2]\ (kd)^2\ q^s/(2d). \tag{3}$$

Assuming a harmonic wave, with space and time dependence expressed by $\exp i(\omega t-kx)$, where the x-coordinate is in the direction of the wave parallel to the plate's surface, and introducing a new variable τ by $k^2 = k_o{}^2(1+\tau^2)$, Eq.(2) is replaced by an algebraic equation in terms of τ. For fluid loading on both sides one has $q^s = 2\ \omega^2\ \rho_o\ w\ /(k_o\ \tau)$ and Eq.(2) becomes

$$\tau^3 - \Omega\ \tau^2 + \tau\ [1-(c_o/c_p)^2]\ - \Omega\ [1-(c_o/c_d)^2] = 0, \tag{4}$$

where a characteristic frequency ω_c and a corresponding dimensionless frequency Ω are introduced by $\omega_c = (\rho/\rho_o)\ c_p{}^2/(c_o\ d)$ and $\Omega = \omega/\omega_c$.

The characteristic frequency ω_c for brass in water is 1.36×10^6 rad/s, and thus for the frequency region where thin-plate theory may be applied, the non-dimensional frequency Ω is very small. As a consequence there exists a small real root, for small Ω, namely

$$\tau \sim \Omega \frac{1 - (c_o/c_d)^2}{1 - (c_o/c_p)^2} . \tag{5}$$

This root corresponds to a non-radiating wave in the plate, with a constant speed slightly less than the speed in the medium, (for low frequency).

By division, one determines the quadratic equation for the two remaining complex conjugate roots,

$$\tau^2 + a\,\tau + b = 0, \tag{6}$$

$$\text{where} \quad a = \Omega \frac{(c_o/c_p)^2 - (c_o/c_d)^2}{1 - (c_o/c_p)^2} \quad \text{and} \quad b = 1 - (c_o/c_p)^2$$

One of these gives a wave number with positive attenuation constant α.

In Fig. 1 a comparison is shown of the relative attenuation α/k', according to thin-plate theory and to exact elasticity theory, first for two-sided fluid loading. Although the boundary conditions for extensional waves cannot be satisfied for one-sided fluid loading without additional flexural waves, one might take half the value of q^s as given before, and carry this through the analysis. The results in Fig. 1 show that the attenuation for one-sided fluid-loading thus computed does not compare well with the exact-elasticity result.

CONSTRAINED PLATE IN VACUUM

In Fig. 2, a threefold comparison is shown for the relative attenuation constant for extensional waves propagating in a constrained plate, without fluid loading, for three models. The sharp dip in attenuation near 15 kHz is due to an "equivoluminal mode", whereby the tangential velocity component at the faces of the plate is zero, thus no shear exists in the elastomer layer. Of course this feature does not show up in the extended Kerwin model.

FLUID-LOADED, CONSTRAINED PLATE

Flexural waves

Various typical features for damping of flexural waves by radiation and viscoelastic effects may be seen in Fig. 3. One sees that in the high-frequency range the total attenuation is mostly due to radiation. In the middle range the attenuation is due to viscoelastic damping in the elastomer. At the low frequency end a feature appears of high damping due to radiation. It may be pointed out that this is a consequence of the infinite extent of the plate; for finite plates this "radiation" would not be expected to be found in the farfield. See Ref. [3] for further details.

Extensional waves

In Fig.4, a comparison is shown for the relative attenuation constant as a function of frequency for a constrained plate loaded by fluid on both sides, on the side of the added layers, and on the opposite side, computed by exact elasticity theory. One sees that for a large frequency region there is not much difference between the curves, except for a factor of two between the double-sided and one-sided fluid loading. At the low frequency end the curves for the two cases part, while at the high frequency side two of the curves converge and the third one follows a different path. If one compares this figure with Fig.1, one sees that the damping due to the elastomer layer does not make much difference in the total attenuation, except for a different structure at the high-frequency end.

In all the various configurations for extensional waves studied here, there is little variation of the phase speed from that for a single, unloaded plate. This is quite different from the behavior of the phase speed for flexural waves at high frequencies [3].

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4. M.C. Junger and D. Feit, Sound, Structures, and Their Interaction, MIT, Cambridge, 2nd ed., 1986, Secs. 8.1 and 8.2
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ACKNOWLEDGEMENT

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TABLE I. Material and geometric parameters and derived quantities.

Base plate, brass		Elastomer layer (hypothetical)	
h_1 = 10 cm	E_1 = 104 GPa	h_2 = 1.24 mm	ρ_2 = 1100 kg/m^3
ν_1 = 0.37	ρ_1 = 8500 kg/m^3	G_2 = 10 MPa	β = 1.0
G_1 = 38 GPa	c_p = 3765 m/s	Bulk modulus = 1.0 GPa	
ω_c = 1.63 Mrad/s		Constraining layer, aluminum	
Fluid, water		h_3 = 2.48 mm	E_3 = 71 GPa
ρ_o = 998 kg/m^3	c_o = 1481 m/s	ν_3 = 0.33	ρ_3 = 2700 kg/m^3

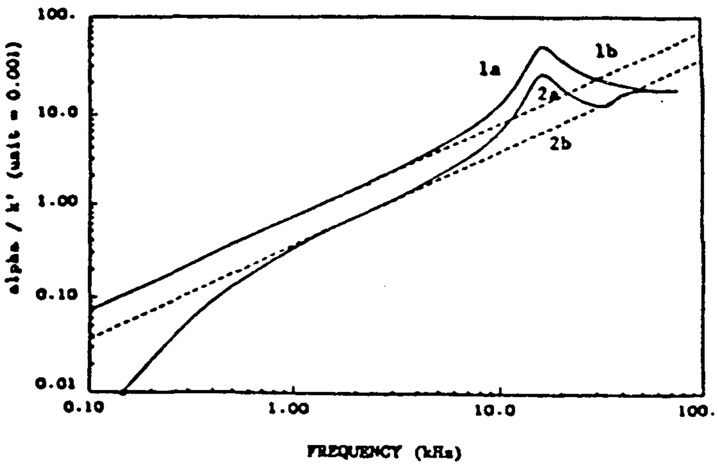

Figure 1. Extensional wave; single, fluid-loaded plate. Fluid on:
1-both sides; a-exact b-thin plate
2-one side; a-exact b-thin plate

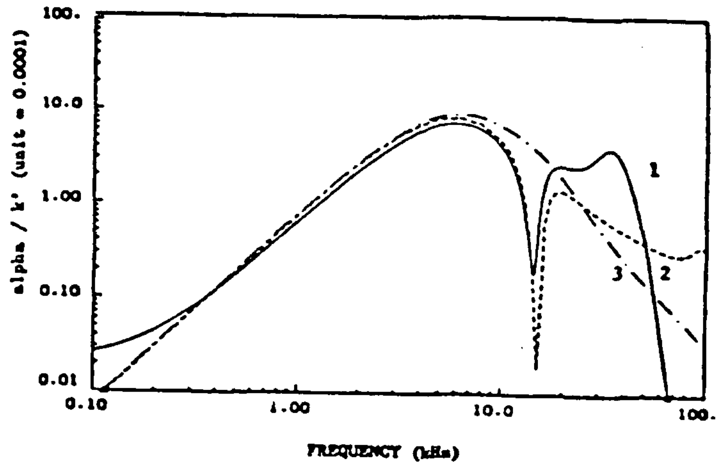

Figure 2. Extensional wave; constrained plate in vacuum. Model: 1-exact; 2-hybrid; 3-extended Kerwin

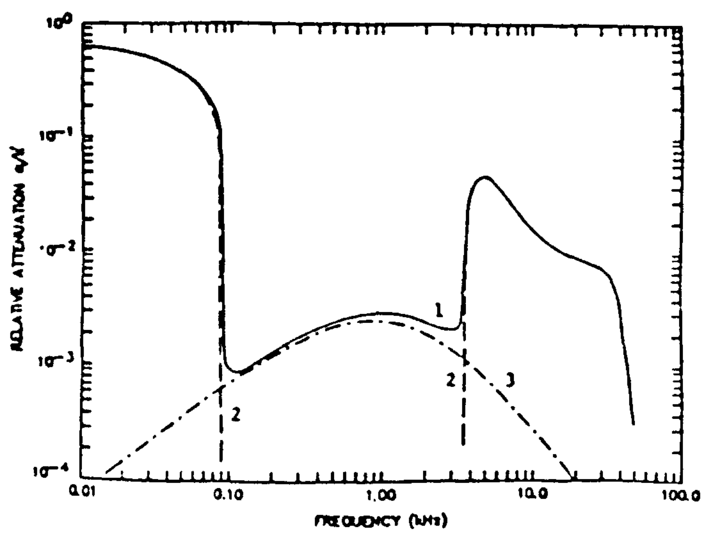

Figure 3. Flexural wave; exact model. 1-constrained plate, water on elastomer side; 2-single plate, water on one side; 3-constrained plate in vacuum

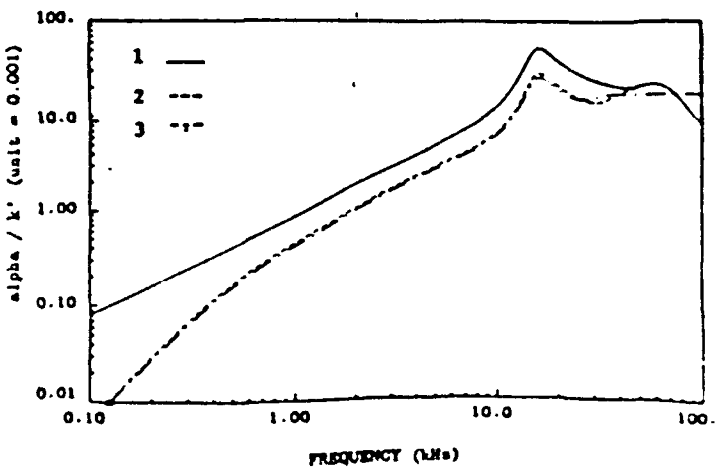

Figure 4. Extensional wave, constrained plate. 1-water on both sides; 2-water on elastomer side; 3-water on opposite side. Exact model.